YEAR 11
TERM 4 EXAMINATION 2010

MATHEMATICS
EXTENSION 1

Time Allowed *– 90 minutes*
(Plus 5 minutes Reading Time)

All questions may be attempted

All questions are of equal value

Department of Education approved calculators are permitted

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each question must show your Candidate's Number.

Question 1 (9 Marks) **Marks**

(a) Given $A(-7,1), B(-4,6), C(1,3)$ and $D(-2,-2)$:

i. Prove that AC and BD are equal in length. **1**

ii. Hence or otherwise prove that $ABCD$ is a square. **2**

(b) Evaluate $\text{cosec}\left(\frac{-5\pi}{6}\right)$. **1**

(c) Find $\int (5-4x)^6 dx$. **1**

(d) Find the equation for y, given that $y' = \frac{x+1}{x}$ and y passes through $(1,5)$. **2**

(e) Solve for x: $\frac{2x}{1-x} < 0$. **2**

Question 2 (9 Marks) – START A NEW PAGE

(a) Find the equation of the inflexional tangent of $y = x^3 - 2x^2 - 2x + 2$. **3**

(b) Using the substitution $u = x^3 - 2$, evaluate the integral $\int_2^3 x^2\sqrt{x^3-2}\,dx$. **3**

(c) The strength s of a rectangular beam is in proportion to the product of its width w and the square of its depth d, that is, $s = kwd^2$ for some positive constant k. Find the dimensions of the strongest rectangular beam that can be cut from a cylindrical log of diameter 48cm. **3**

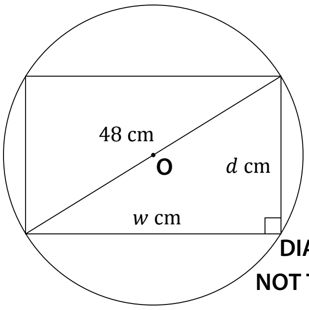


DIAGRAM NOT TO SCALE

Question 3 (9 Marks) – START A NEW PAGE

(a) Determine the minimum value of $y = \frac{x+1}{x^2}$. **3**

(b) Use Simpson's rule with 3 function values to estimate $\int_{-1}^{3} \sqrt{x+3}\ln\sqrt{x^2+1}\,dx$ correct to 2 decimal places. **2**

(c) Given $y = \frac{\tan x}{\sqrt{3}} - 1$:

i. Prove that $(0,-1)$ is a point of inflexion. **2**

ii. Hence, neatly sketch the curve for $0 \le x \le 2\pi$. **2**

Question 4 (9 Marks) – START A NEW PAGE **Marks**

(a) Find $\int \frac{2x^2-6}{x^2+9}\,dx$. **2**

(b) By considering the equation $x^2 + y^2 = r^2$, prove that the volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$. **3**

(c) The region bounded by the curve $y = \dfrac{1}{x}$ and the x-axis between $x = 1$ and $x = 2$ is rotated through one complete revolution about the y-axis. Find the exact volume of the solid of revolution so formed. **4**

Question 5 (9 Marks) – START A NEW PAGE

(a) If $f(x) = \sin^{-1} x$ & $g(x) = \cos^{-1} x$, prove that $\frac{d}{dx}[f(x)g(x)] = -f'(x)[f(x) - g(x)]$. **2**

(b) A function is defined by $f(x) = \dfrac{\sqrt{4-x^2}}{x}$.

i. Find the largest positive domain for which $y = f(x)$ has an inverse. **1**

ii. For the domain in (i), find an expression for $y = f^{-1}(x)$, the inverse of $y = f(x)$. **2**

iii. State the domain and range of $y = f^{-1}(x)$. **2**

iv. For the domain in (i) and on the same graph, neatly sketch $y = f(x)$ and $y = f^{-1}(x)$, clearly identifying each sketch. **2**

EXAM CONTINUES OVERLEAF

Question 6 (9 Marks) – START A NEW PAGE **Marks**

(a) i. Express $3\sin x + \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 2\pi$. **2**

ii. Hence find the general solutions to $3\sin x + \sqrt{3}\cos x = 3$. **2**

(b) $ABCD$ is a square with sides of length 1m. Quadrants CBD and APQ touch at point X. CP and CQ are constructed such that the shaded area is enclosed by figure $RQXPS$.

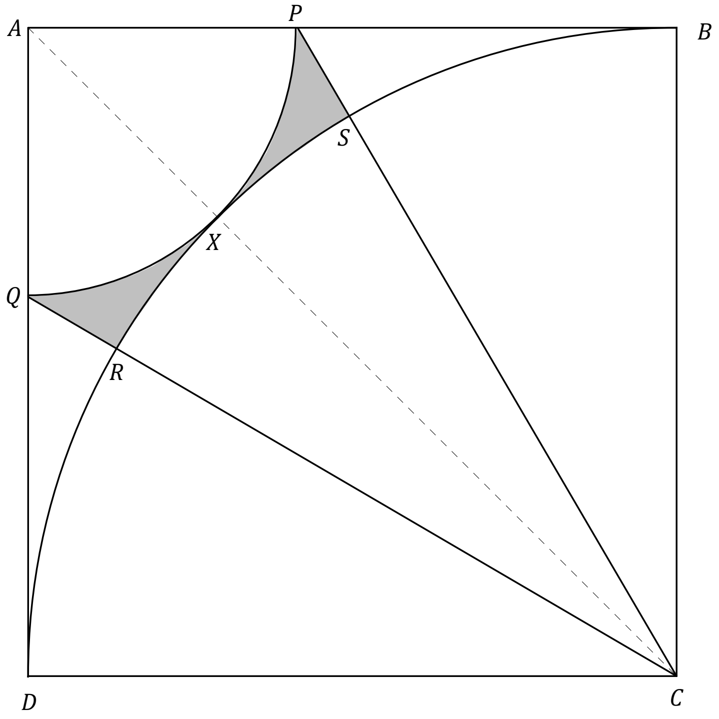


DIAGRAM NOT TO SCALE

i. Show that $\angle PCQ = 0.51$ radians (to 2 decimal places). **2**

ii. Hence or otherwise calculate the shaded area to the nearest square centimetre. **3**

Question 7 (9 Marks) – START A NEW PAGE

(a) Find the equation of the normal to the curve $y = \sin^{-1}\left(\frac{\sqrt{x}}{2}\right)$ where $x = 1$. **3**

(b) For the function $y = \tan^{-1}(2x) + \tan^{-1}\left(\frac{2}{x}\right)$:

i. Show that $\frac{dy}{dx} = \frac{2}{4x^2+1} - \frac{2}{4+x^2}$. **1**

ii. State the domain and range of the function. **2**

iii. Neatly sketch the graph of $y = \tan^{-1}(2x) + \tan^{-1}\left(\frac{2}{x}\right)$, clearly labelling all stationary points. **3**

END OF EXAM

Y12 M.EXT1 ASSESS TEST 1

TERM4, 2010

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
a i) $AC = \sqrt{(-7-1)^2+(1-3)^2} = \sqrt{8^2+2^2} = \sqrt{68}$ $BD = \sqrt{(-4+2)^2+(6--2)^2} = \sqrt{2^2+8^2} = \sqrt{68}$ $\therefore AC = BD$	1	
ii) $m(Ac) = \frac{3-1}{1--7} = \frac{2}{8} = \frac{1}{4}$ $m(BD) = \frac{6--2}{-4--2} = \frac{8}{-2} = -4$ $\therefore m(Ac) \times m(BD) = -1 \quad Ac \perp BD$	1	
$mid(Ac) = \left(\frac{-7+1}{2}, \frac{1+3}{2}\right) = (-3, 2)$ $mid(BD) = \left(\frac{-4-2}{2}, \frac{6-2}{2}\right) = (-3, 2)$ diagonals are equal and bisect each other at 90°	$\}$ 1	most students forgot to prove <u>bisect</u> only 1 m must give reason $-\frac{1}{2}$m
b) $\frac{1}{\sin\left(\frac{-5\pi}{6}\right)} = \frac{1}{-\frac{1}{2}} = -2$	1	$\}$ no half mark. Overall well done.
c) $\frac{(5-4x)^7}{-28} + c$	1	
d) $y = \int 1 + \frac{1}{x}\, dx = x + \ln x + c$ $5 = 1 + \ln 1 + c$	1	
$c = 4$ $y = x + \ln x + 4$	1	forgot last line $-\frac{1}{2}$m
c) $\frac{2x}{1-x} \times (1-x)^2 < 0(1-x)^2$ $2x(1-x) < 0$ 	1	$x<0$ or $x>1$ use 'and' $-\frac{1}{2}$m
$\therefore x < 0$ or $x > 1$	1	

YR 11 TERM 4

MATHEMATICS Extension 1: Question 2 2010

Suggested Solutions	Marks	Marker's Comments
(c) $d^2+w^2=48^2$ (Pythagoras theorem) $d^2=48^2-w^2$ $\therefore\ S=kw(48^2-w^2)$ $S=48^2kw-kw^3$ $\therefore\ \frac{dS}{dw}=48^2k-3kw^2$ $=k(48^2-3w^2)$	$\frac{1}{2}$	Not squaring 48 made the calculations a lot easier, so a max. of 2 marks for the whole question.
Stationary pts exist when $\frac{dS}{dw}=0$ i.e. $k(48^2-3w^2)=0$ $3w^2=48^2$ $w^2=\frac{48^2}{3}$ $w=\frac{48}{\sqrt{3}}$ (as $w>0$) $w=16\sqrt{3}$ cm ($\sqrt{768}$)	$\frac{1}{2}$	$\frac{1}{2}$ mark deducted for not stating $w>0$ & $d>0$ Locating the stat. pt.
Test nature $\frac{d^2S}{dw^2}=-6kw$ when $w=16\sqrt{3}$, $\frac{d^2S}{dw^2}=-96\sqrt{3}k$ Now for $k>0$ $\frac{d^2S}{dw^2}<0$ {concave down $\therefore$ Local maximum exists when $w=16\sqrt{3}$ cm	1	if the students did the 1st derivative test they needed to say $k>0$ for the full mark. (Strength is positive)
As the function is continuous for $0\le w\le 48$ and there is only one turning pt. the local max. is also the absolute max.		$\frac{1}{2}$ mark deducted for not proving absolute max.
When $w=16\sqrt{3}$, $d^2=48^2-(16\sqrt{3})^2$ $d^2=1536$ $d=16\sqrt{6}$ cm ($d>0$) $\therefore$ Dimensions of the strongest rectangular beam are $16\sqrt{3}$ cm wide and $16\sqrt{6}$ cm deep.	1	$\frac{1}{2}$ mark deducted for no units

YR 11 TERM 4 **MATHEMATICS Extension 1: Question** 2 2010

Suggested Solutions	**Marks**	**Marker's Comments**
Qu 2		
(a) $y=x^3-2x^2-2x+2$		
$y'=3x^2-4x-2$		
$y''=6x-4$		
For inflexion pts $y''=0$		
$6x-4=0$		
$x=\frac{2}{3},\ y=\frac{2}{27}$	1	
$\begin{array}{\|c\|c\|c\|c\|}\hline x & \frac{1}{3} & \frac{2}{3} & 1 \\ \hline y'' & -2 & 0 & 2 \\ \hline\end{array}$ Change in concavity ∴ Inflexion pt. at $\left(\frac{2}{3}, \frac{2}{7}\right)$	$\frac{1}{2}$	$\frac{1}{2}$ mark deducted for not testing for inflexion pt.
When $x=\frac{2}{3},\ y'=-\frac{10}{3}$	$\frac{1}{2}$	
Eqn. of inflexional tangent is:		
$y-\frac{2}{27}=-\frac{10}{3}\left(x-\frac{2}{3}\right)$		
$y-\frac{2}{27}=-\frac{10x}{3}+\frac{20}{9}$		
$27y-2=-90x+60$		$y=-\frac{10}{3}x+2\frac{8}{27}$
$90x+27y-62=0$	1	$y=-\frac{10x}{3}+\frac{62}{27}$
(b) $\int_2^3 x^2\sqrt{x^3-2}\,dx=\int_6^{25}\frac{1}{3}\sqrt{u}\,du$	1	1 mark for correct substitution
Let $u=x^3-2$		
$\frac{du}{dx}=3x^2$		1 mark for the change of limits
$du=3x^2dx$		
When $x=2$ $u=6$		
$x=3$ $u=25$	1	
$\therefore \int_6^{25}\frac{1}{3}\sqrt{u}\,du=\frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_6^{25}$		
$=\frac{2}{9}\left[u^{\frac{3}{2}}\right]_6^{25}$		
$=\frac{2}{9}(125-6\sqrt{6})$	1	1 mark for correct answer
OR $=\frac{250-12\sqrt{6}}{9}$		
OR $\doteqdot 24.51$ (2dp)		

Y12. M. EXT 1. ASSESSMENT TEST 1
TERM 4, 2010

MATHEMATICS Extension 1 : Question 3

Suggested Solutions | **Marks** | **Marker's Comments**

Q3(a) $y = \frac{x+1}{x} = x^{-1} + x^{-2},\ x \neq 0$

$\frac{dy}{dx} = -\frac{1}{x^2} - \frac{2}{x^3} = -\frac{(x+2)}{x^3}$ ½

$\frac{d^2y}{dx^2} = \frac{2}{x^3} + \frac{6}{x^4} = \frac{2x+6}{x^4}$

For possible TPs to occur $\frac{dy}{dx} = 0$

$\therefore -\frac{1}{x^2} - \frac{2}{x^3} = -\frac{(x+2)}{x^3} = 0$

$\therefore \underline{x = -2}\quad \underline{y = -\frac{1}{4}}\qquad SP = (-2, -\frac{1}{4})$ ½ ea

TEST: at $x = -2$

- $\frac{d^2y}{dx^2} = -\frac{2}{8} + \frac{6}{16} = \frac{1}{8} > 0$

$\therefore$ concave upwards

$\therefore$ a Rel. min TP at $x = -2$

x	-3	-2	-1.9	-1
$\frac{dy}{dx}$	$-\frac{1}{27}$ -0.037	0	0.0145	1
	⊖		⊕	
	\	—	/	

Since y is cont. and differentiable for $x < 0$ or $x > 0$ and there is only one TP

$\therefore$ absolute min is $-\frac{1}{4}$ (when $x = -2$) ½

OR: $x^2y = x + 1$

$x^2y - x - 1 = 0$

For x to be real $\Delta \geq 0$ Df

$\therefore (-1)^2 - 4 \times y \times (-1) \geq 0$

$1 + 4y \geq 0$

$y \geq -\frac{1}{4}$

$\therefore R_f = \{y : y \geq -\frac{1}{4}\}$

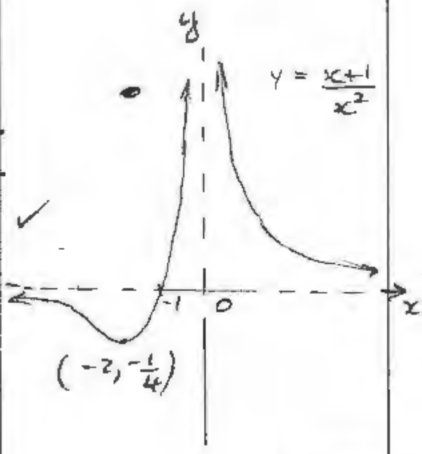
when $y = -\frac{1}{4}$ there is a TP at $x = \frac{-(-1)}{2 \times (-\frac{1}{4})} = -2$

$\therefore$ min (abs) value is $-\frac{1}{4}$ when $x = -2$

Q. Rule gives

$\frac{1x^2 - (x+1) \times 2x}{x^4} = \frac{-x^2 - 2x}{x^4}$

$=$



$\boxed{3}$

$x = -\frac{b}{2a}$

(b) $\int_{-1}^{3} \sqrt{x+3}.\ln\sqrt{x^2+1}\,dx$: $h = \frac{3-(-1)}{2} = \frac{4}{2} = 2$ ½

$\doteqdot \frac{h}{3}[Y_1 + 4Y_2 + Y_3]$

x	-1	1	3
Y	$\sqrt{2}\ln\sqrt{2}$ 0.4901...	$2\ln\sqrt{2}$ 0.6931...	$\sqrt{6}\ln\sqrt{10}$ 2.8201...
	Y_1	Y_2	Y_3

$= \frac{2}{3}[\sqrt{2}\ln\sqrt{2} + 4 \times 2\ln\sqrt{2} + \sqrt{6}\ln\sqrt{10}]$

$= \frac{2}{3}[6.082797...] = 4.055198052...$

$= \underline{4.06}$ (2 d.p)

$\frac{b-a}{6} = \frac{3-(-1)}{6} = \frac{2}{3}$

$a = -1$
$b = 3$
$\frac{a+b}{2} = 1$

½ For $\frac{2}{3}$
1 For CNEs Y_1, Y_2, Y_3
½ For 4.06 gained correctly

$\boxed{2}$

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments

(c) (i) $y = \frac{1}{\sqrt{3}}\tan x - 1$

$\frac{dy}{dx} = \frac{1}{\sqrt{3}}\sec^2 x$ (½)

$\frac{d^2y}{dx^2} = \frac{2}{\sqrt{3}}\sec^2 x \tan x = \frac{2}{\sqrt{3}}\frac{\sin x}{\cos^3 x}$. ½

For possible P.O.Is to occur $\frac{d^2y}{dx^2} = 0$

$\therefore \sec^2 x \tan x = 0$ i.e. $\tan x = 0$

$\therefore x = 0, \pi, 2\pi, \ldots$

$y = -1, -1, -1 \ldots$

$\therefore$ poss PoI $(0, -1)$

or subst $x = 0$ in $\frac{d^2y}{dx^2}$ to get $y'' = 0$

$y'' = \frac{2}{\sqrt{3}}\sec^2 x . \tan x$

TEST

x	-1	0	$0\cdot1$	$\frac{1}{2}$	1	$\frac{\pi}{4}$
$\frac{d^2y}{dx^2}$	$-6\cdot16$ ⊖	0	$0\cdot117$	$0\cdot819$	$6\cdot16$	$\frac{4}{\sqrt{3}}$

⊕ ✓ 1

x	0^-	0	0^+
y''	$\frac{2}{\sqrt{3}}(+)(-)$	0	$\frac{2}{\sqrt{3}}(+)(+)$

1 For Test

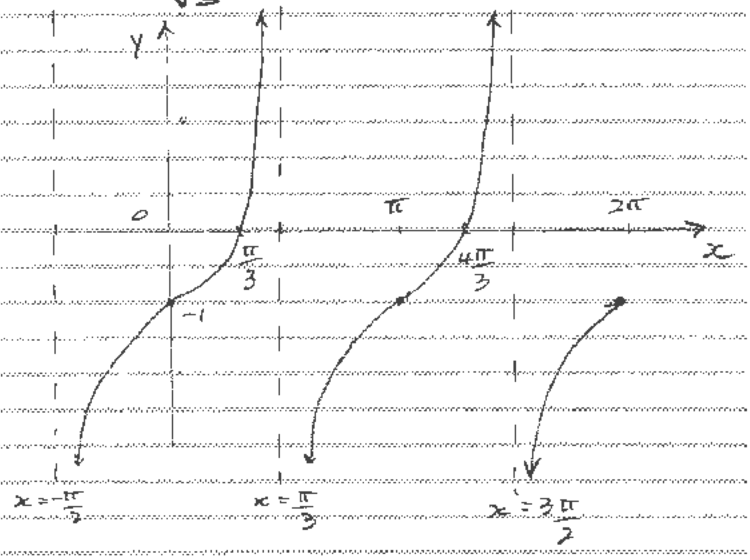
$\therefore$ a change in sign

$\therefore$ a change in concavity $(- \ 0 \ +)$

$\therefore$ a Point of inflection at $(0, -1)$

$\boxed{2}$

(ii) sketch $y = \frac{1}{\sqrt{3}}\tan x - 1$ $\quad -\frac{\pi}{2} \le x \le 2\pi$



½ For x-int $\frac{\pi}{3}$ or $\frac{4\pi}{3}$

½ PoI at $(0, -1)$

½ V.A $x = \frac{\pi}{2}$; $\frac{3\pi}{2}$

½ shape

$\boxed{2}$

Year 11 Assessment 1 Term 4 2010 – Ext 1 Marking Scheme – Q4 – L.Kim

(a) $\int \frac{2x^2-6}{x^2+9}dx \quad = 2\int \frac{x^2+9-12}{x^2+9}dx$

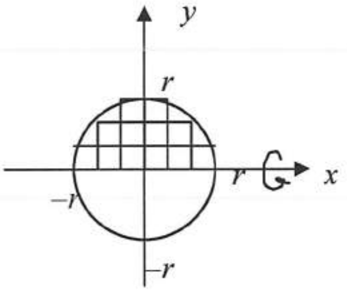
$= 2\int 1dx - \int \frac{12}{x^2+9}dx$ ☑

$=2[x - \ 4\tan^{-1}\left(\frac{x}{3}\right)] + c,$ ☑

where c is a constant

- Correctly breaks up the fraction – 1 mk
- Correctly integrates both– 1 mk
- Correctly integrating $\int \frac{1}{x^2+9}dx$ - ½ mark only

(b) $x^2 + y^2 = r^2$ → circle centre (0,0) radius r.



Take shaded area and rotate it around the x – axis

$\therefore$ volume = $\pi \int_{-r}^{r} y^2dx$

$= \pi \int_{-r}^{r} \left(r^2 - x^2\right)dx$ ☑

$= \pi \left[r^2x - \frac{x^3}{3}\right]_{-r}^{r}$ ☑

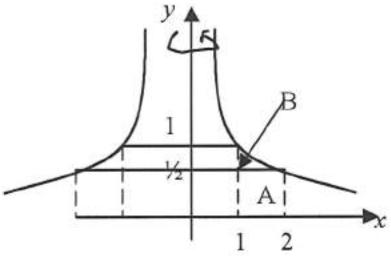
$= \pi \left[r^2(r) - \frac{(r)^3}{3} - r^2(-r) + \frac{(-r)^3}{3}\right]$

$= \pi \left[2r^3 - \frac{2r^3}{3}\right] = \frac{4\pi r^3}{3}$ as required

☑

- Students can do rotation about the y – axis also.
- Correctly uses the equation of circle to get to correct definite integral – 1 mk
- Correctly integrates wrt x – 1 mk
- Correctly substitutes and simplifies <u>clearly</u> showing how they get answer – 1 mk

(c)



To find this volume you must add the Vol A to Vol B

$\therefore$ Volume A = Vol. cylinder

$$= \pi\left(2^2\right)\left(\frac{1}{2}\right) - \pi\left(1^2\right)\frac{1}{2} = \frac{3\pi}{2}$$

$$\text{Volume B} = \pi\int_{\frac{1}{2}}^{1} \frac{1}{y^2}dy - \pi\left(1^2\right)\frac{1}{2}$$

$$= \pi\left[-\frac{1}{y}\right]_{\frac{1}{2}}^{1} - \frac{\pi}{2}$$

$$= -\pi + 2\pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

$\therefore$ total volume = $\dfrac{3\pi}{2} + \dfrac{\pi}{2} = 2\pi$ cubic units

TOTAL = <u>9 MARKS</u>

- This question was very poorly done.
- Most students had no idea what to do with the question.

- Correctly finds vol. A 1 ½ mks
- Correctly integrates $\pi\int_{\frac{1}{2}}^{1} \frac{1}{y^2}dy$ - 1 ½ mks
- Correctly subtracts off the vol. of cylinder – ½ mk
- Final answer – ½ mk

NOTES:

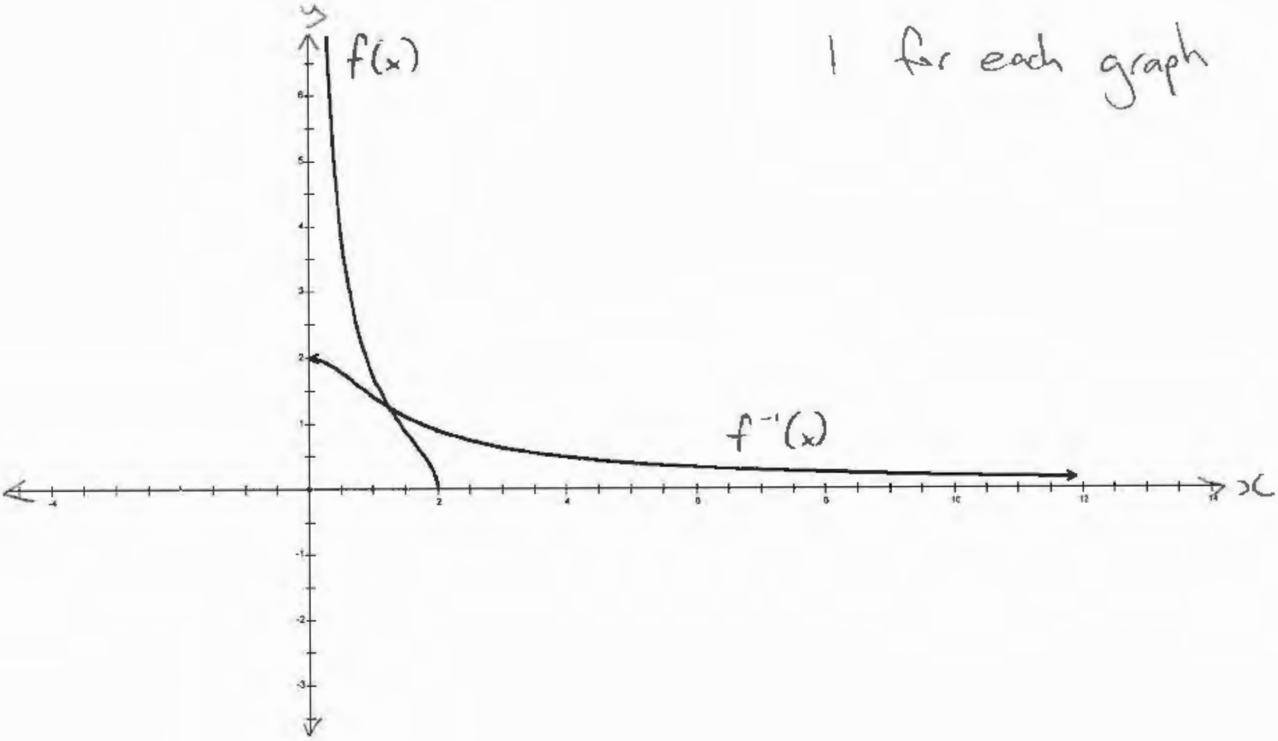
1. Max 1 mk if correctly did rotation about x – axis i.e.

$$\pi\int_{1}^{2} \frac{1}{y^2}dx$$

2. Also some students only did $\pi\int_{1}^{2} \frac{1}{y^2}dy$ - again max 1 mk.

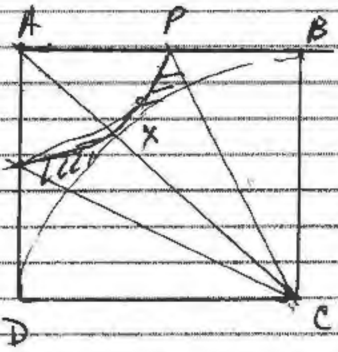
MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
(a) $y = \sin^{-1}x . \cos^{-1}x$		
$f'(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad g'(x) = \frac{-1}{\sqrt{1-x^2}}$	1/2	
$\frac{dy}{dx} = \sin^{-1}x \frac{-1}{\sqrt{1-x^2}} + \frac{\cos^{-1}x}{\sqrt{1-x^2}}$	1/2	
$= \frac{-1}{\sqrt{1-x^2}}\left[\sin^{-1}x + \cos^{-1}x\right]$	1/2	
$= -f'(x)\left[f(x) - g(x)\right]$	1/2	
(b) (i) Domain: $0 < x \leq 2$	1/2 for "<" 1/2 for 0 and 2	
(ii) $y = \frac{\sqrt{4-x^2}}{x}$		
$x = \frac{\sqrt{4-y^2}}{y}$	1/2	
$xy = \sqrt{4-y^2}$		
$x^2y^2 = 4 - y^2$		
$x^2y^2 + y^2 = 4$	1/2	
$y^2(x^2+1) = 4$		
$y^2 = \frac{4}{x^2+1}$		
$y = \frac{\pm 2}{x^2+1}$	1/2	
$y = \frac{2}{\sqrt{x^2+1}}$ but Range is $0<y\leq 2$ for the inverse function!!		1/2
(iii) D: $x > 0$	1	lots of students wrote the domain and range wrong but then managed to graph the correct graph!?!??....
R: $0 < ~~0~~ y \leq 2$	1	
(iv)		



1 for each graph

Y11 3U T4 **MATHEMATICS Extension 1 : Question……** 6

Suggested Solutions	**Marks**	**Marker's Comments**
a) $R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha = 3\sin x + \sqrt{3}\cos x$.		
$\therefore R\sin\alpha = -\sqrt{3}$ ① $\quad R > 0 \quad \sin\alpha < 0$		
$R\cos\alpha = 3$ ② $\quad \cos\alpha > 0$.		
$\tan\alpha = \frac{-1}{\sqrt{3}} \quad \frac{3\pi}{2} < \alpha < 2\pi$		
$\alpha = \frac{11\pi}{6}$.	1	$-R$ ✱ right or wrong.
$R = \sqrt{3^2+(\sqrt{3})^2} =$		
$= \sqrt{12}$	1	$-\alpha$ ✱
$= 2\sqrt{3}$		
$\therefore 3\sin x + \sqrt{3}\cos x = 2\sqrt{3}\sin\left(x - \frac{11\pi}{6}\right)$		
(ii) $\therefore 2\sqrt{3}\sin\left(x - \frac{11\pi}{6}\right) = 3$		
$\sin\left(x - \frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$		$\frac{1}{2}$ for $\frac{\pi}{3}$.
$x - \frac{11\pi}{6} = \pi k + (-1)^k . \frac{\pi}{3}$	1	
$x = \frac{11\pi}{6} + \pi k + (-1)^k . \frac{\pi}{3}$ k integer	1	
(b) 		
$AC = \sqrt{1^2+1^2}$ (Pythagoras		
$= \sqrt{2}$		
$\therefore AX = \sqrt{2}-1$		
$\therefore AP = \sqrt{2}-1$		
$DQ = 1 - (\sqrt{2}-1)$		
$= 2-\sqrt{2}$		
$\tan\angle QCD = 2-\sqrt{2}$	1	
$\angle QCD = \tan^{-1}(2-\sqrt{2})$		
$\angle PCQ = \frac{\pi}{2} - 2\tan^{-1}(2-\sqrt{2})$	1	
$= 0.51$ rad.		
Area $APCQ = 2\lvert APC\rvert$		
$= 2\times\frac{1}{2}(\sqrt{2}-1).1$	1	
$= \sqrt{2}-1 \quad (0.41421)$		
Area quad $APQ = \frac{1}{2}.(\sqrt{2}-1)^2.\frac{\pi}{2} = 0.13475$.	1	
Area sector $ORS = \frac{1}{2}.1^2.0.51 = 0.25550$.	$\frac{1}{2}$	
Total = Area APCQ − Area quadrant − Area Sector		
$= 0.41421 - 0.13475 - 0.2550$		
$= 0.02396$	$\frac{1}{2}$	

$= 0.0240 \ (m^2)$

OR $240\ cm^2$ (nearest cm^2).

MATHS EXT 1 : TERM 4 Yr 11 Task 1 : 2010

Q7

(a) $y = \sin^{-1}(\frac{\sqrt{x}}{2})$

$$y' = \frac{1}{\sqrt{1-(\frac{\sqrt{x}}{2})^2}}.\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{4\sqrt{x}\sqrt{1-\frac{x}{4}}}$$

$$= \frac{1}{4\sqrt{x}.\frac{1}{2}\sqrt{4-x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{4-x}}$$

$$\therefore m_{tangent} = y'(1) = \frac{1}{\sqrt{12}}$$

$$\therefore m_{normal} = -\sqrt{12}$$

When $x = 1,\ y = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Hence equation of normal is

$$y - \frac{\pi}{6} = -\sqrt{12}(x-1)$$

$$\Rightarrow y = -\sqrt{12}x + (\sqrt{12} + \frac{\pi}{6})$$

$\frac{1}{2}$ for derivative (in unsimplified form)

$\frac{1}{2}$ for slope of tangent

$\frac{1}{2}$ for slope of normal

$\frac{1}{2}$ for coordinates of point of contact

$\frac{1}{2}$ for m and coordinates in point gradient formula

$\frac{1}{2}$ for expressing equation either in the form $y = mx + b$ or $ax + by + c = 0$

Note: (i) If $m_{tangent}$ used to get equation, then max $2\frac{1}{2}$

(ii) If derivative incorrect, then max $2\frac{1}{2}$

(iii) If no working for $\pi/6$ minus $\frac{1}{2}$

(iv) Minus $\frac{1}{2}$ for poor notation eg m_N, m_T, $\perp y'$, perpendicular -1 etc

(b) (i) $y = \tan^{-1}\frac{x}{\frac{1}{2}} + \tan^{-1}\frac{2}{x}$

$$\frac{dy}{dx} = \frac{1}{1+(2x)^2}.2 + \frac{1}{1+\left(\frac{2}{x}\right)^2}.(-2x^{-2})$$

$$= \frac{2}{1+4x^2} - \frac{2}{x^2(1+\frac{4}{x^2})}$$

$$= \frac{2}{1+4x^2} - \frac{2}{x^2+4}$$

½ for both terms of derivative

½ for simplification - must show how.

(ii) Domain: $x \neq 0, x \in$ ℝeals and

Range: $\frac{\pi}{2} < y \leq 2.21 \quad \cup -2.21 \leq y < -\frac{\pi}{2}$, $y \in$ ℝeals

$x \neq 0$ or $x < 0$ and $x > 0$ then 1 mark

½ for $x \in \mathbb{R}$ only

0 for $y \in \mathbb{R}$ only.

½ for each range

(iii) $\frac{dy}{dx} = 0$ …… for stationary points:

i.e.

$$\frac{2}{1+4x^2} - \frac{2}{x^2+4} = 0$$

$\Rightarrow$

$$\frac{2}{1+4x^2} = \frac{2}{x^2+4}$$

$x^2 + 4 = 1 + 4x^2$

$x = \pm 1$

$when\ x = 1, y = 2.21\ or\ \ 2\tan^{-1} 2$

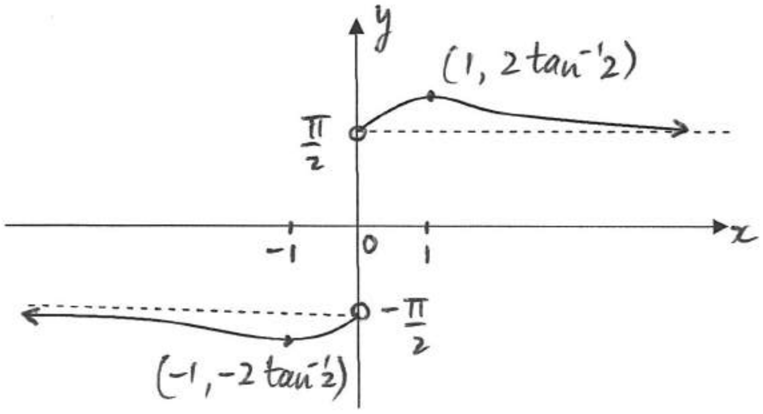
$when\ x = -1, y = -2.21\ or\ \ 2\tan^{-1}(-2)$

½ for stationary points

x	-1.5	-1	-0.5	0.5	$+1$	1.5
y'	-3/25	0	9/17	9/17	0	-3/25
	\	—	/	/	—	\

} ½ for testing

Hence $(1, 2\tan^{-1} 2)$ is a local maximum and $(1, -2\tan^{-1} 2)$ is a local minimum



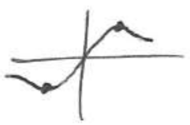
½ for correct intercepts

½ for shape, including discontinuity and asymptote

½ for max on graph

½ for min on graph

max 1½ for



with correct TP + working